The Standard Model for Limping Pedestrians (i.e. Experimentalists)

Philip Bechtle

DESY

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Prerequisites: γ_μ, ∂^μ and the $\dagger$

The notation is a little bit confusing sometimes, so let's try to sort things a little bit:

Fermions are represented by 4-dimensional spinors:

$$\psi(p) = \sqrt{p_0 + m} \begin{pmatrix} \chi_s \\ \frac{\vec{\sigma}\vec{p}}{p_0+m}\chi_s \end{pmatrix}, \quad \chi_{1/2} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \chi_{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

The 4×4 γ matrices are acting on the 4 dimensions of ythe spinors.

An index (γ_μ, A_μ or $F_{\mu\nu}$) always denotes a 4-dimensional Lorentz vector. This 4-dimensional space is independent of the 4-dimensional spinor space.

∂^μ denotes a partial derivative for x^0, x^1, x^2, x^3 respecively.

Einstein convention:
4-vector: x^μ
scalar: $x^\mu y_\mu$
matrix: $x^\mu y^\nu$

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Dirac matrices (each matrix acting on a 4-dim spinor):

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\gamma^5 := i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Hermitean adjoint: $\psi^\dagger$: $a_{ij} = a_{ji}^*$, Dirac adjoint: $\bar{\psi} = \psi^\dagger\gamma^0$

The Lagrangian

Require that the action S remains invariant under small changes of the fiends ϕ:

$$\frac{\delta \mathcal{S}}{\delta \varphi_i} = 0$$

S is determined by the Lagrangian (classically: $\mathcal{L} = T - V$)

$$\mathcal{S}[\varphi_i] = \int \mathcal{L}[\varphi_i(s)]\, \mathrm{d}^n s,$$

where s_α denotes the parameters of the system.

The equations of motion of the system can then be derived from the Euler-Lagrange equation:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi)} \right) - \frac{\partial \mathcal{L}}{\partial \varphi} = 0$$

The Lagrangian

Classical Example in three-dimensional space:

$$L(\vec{x},\dot{\vec{x}}) \;=\; \frac{1}{2}\; m\; \dot{\vec{x}}^2 \;-\; V(\vec{x}).$$

Then, the Euler-Lagrange equation is:

$$\frac{d}{dt}\;\left(\frac{\partial L}{\partial \dot{x}_i}\right)\;-\;\frac{\partial L}{\partial x_i}\;=\;0$$

with $i = 1, 2, 3$. The derivation yields:

$$\frac{\partial L}{\partial x_i} \;=\; -\;\frac{\partial V}{\partial x_i}$$

$$\frac{\partial L}{\partial \dot{x}_i} \;=\; \frac{\partial}{\partial \dot{x}_i}\;\left(\frac{1}{2}\; m\; \dot{\vec{x}}^2\right)\;=\;\frac{1}{2}\; m\;\frac{\partial}{\partial \dot{x}_i}\;(\,\dot{x}_i\,\dot{x}_i\,) = \; m\,\dot{x}_i$$

$$\frac{d}{dt}\;\left(\frac{\partial L}{\partial \dot{x}_i}\right)\;=\; m\,\ddot{x}_i$$

From the Euler-Lagrange-equation we get the equation of motion:

$$m\ddot{\vec{x}} + \nabla V = 0$$

Gauge Transformations

- Global Gauge Invariance:
  Require that $\mathcal{L}$ (i.e. the equation of motion) is invariant under the transformation:

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  with α being the same everywhere.

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This principle is the foundation of the SM

Group Theory in a Tiny Nutshell

A group is a set G (the "underlying set") under a binary operation satisfying three axioms:

- The operation is associative.
- The operation has an identity element.
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Very simple example: 2 is the generator of all numbers 2^n, $n = [0, \inf[$

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Construct the SM particles as elements of a group invariant under operations within the group.

Some Mathematics: $SU(2)$

For the special unitary group $SU(2)$, the generators are proportional to the Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The generators of the group are $\tau_i = \frac{1}{2}\sigma_i$. The Pauli matrices obey

$$\begin{aligned} [\sigma_i, \sigma_j] &= 2i\, \varepsilon_{ijk}\, \sigma_k \\ \{\sigma_i, \sigma_j\} &= 2\delta_{ij} \cdot I \end{aligned}$$

Example for an $SU(2)$ transformation:

$$\psi(x) \to e^{i\tau_i \alpha^i(x)} \psi(x)$$

$SU(2)$ and $SU(3)$ are not abelian, i.e. the generators of the group do not commute.

Some Mathematics: $SU(3)$

The analog of the Pauli matrices for $SU(3)$ are the Gell-Mann matrices:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The generators of $SU(3)$ are defined as T by the relation

$$T_a = \frac{\lambda_a}{2}.$$

Some Mathematics: $SU(3)$

The generators T obey the relations

$$[T_a, T_b] = i \sum_{c=1}^{8} f_{abc} T_c$$

where f is called structure constant and has a value given by

$$f^{123} = 1$$

$$f^{147} = f^{165} = f^{246} = f^{257} = f^{345} = f^{376} = \frac{1}{2}$$

$$f^{458} = f^{678} = \frac{\sqrt{3}}{2}$$

$$\mathrm{tr}(T_a) = 0$$

Introduction: QED

QED is a local abelian $U(1)$ gauge symmetry

Using our knowledge about the Lagrangian, we construct the Lagrangian which gives us the equation of motion of the Dirac equation ($(i\partial_\mu\gamma^\mu - m)\psi = 0$):

$$\mathcal{L}_{\text{free}} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

using $\partial\!\!\!/ = \partial_\mu\gamma^\mu$.
Make the theory gauge invariant under local $U(1)$ transformations:

$$\psi(x) \longrightarrow e^{i\alpha(x)}\psi(x)$$

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That's not invariant!
But luckily it's also not QED . . .

Introduction: QED

In order to save QED under the transformation $U(x) = e^{-1\alpha(x)}$, add a gauge field obeying:

$$A_\mu(x) \rightarrow U^{-1} A_\mu U + \frac{1}{q} U^{-1} \partial_\mu U = A_\mu(x) - \frac{1}{q} \partial_\mu \alpha(x)$$

A miracle has occured: we introduced not only a gauge field, but also a charge q. Also, we would have needed the photon A_μ anyway . . .

Now modify the derivative:

$$\partial_\mu \rightarrow \partial_\mu + iqA_\mu(x) = D_\mu$$

Let's write $\mathcal{L}$ again with all possible Lorentz and gauge invariant terms:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi - q\bar{\psi}\not{A}\psi$$

using

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Introduction: QED

Let's check the transformational behaviour under local U(1) again:

$$\mathcal{L} \rightarrow \mathcal{L}' = -\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \bar{\psi}'(i\not{\partial} - m)\psi' - q\bar{\psi}'\not{A}'\psi'$$

$$= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\not{\partial} - m)\psi - \bar{\psi}\gamma_\mu\psi(\partial^\mu\alpha(x)) - q\bar{\psi}\gamma_\mu\psi A^\mu + \bar{\psi}\gamma_\mu\psi(\partial^\mu\alpha(x))$$

$$= \mathcal{L}$$

with

$$F'_{\mu\nu} = \partial_\mu(A_\nu - \frac{1}{q}\partial_\nu\alpha(x)) - \partial_\nu(A_\mu - \frac{1}{q}\partial_\nu\alpha(x))$$

$$= F_{\mu\nu} - \partial_\mu\frac{1}{q}\partial_\nu\alpha(x) + \partial_\nu\frac{1}{q}\partial_\mu\alpha(x) = F_{\mu\nu}$$

QED including a gauge field is invariant under local U(1)!

Use this principle to construct the SM

QCD: $SU(3)_C$

The fundamental states of QCD are the three color states of the quarks:

$$q = \begin{pmatrix} q_R \\ q_G \\ q_B \end{pmatrix},$$

which are transforming under the fundamental representation of $SU(3)_C$:

$$q_i \to q_i{}' = \left(e^{i\alpha^a(x)\frac{\lambda_a}{2}}\right)_{ij} q_j,$$

where λ_a with $a = 1, \ldots, 8$ are the eight 3×3 Gell-Mann-Matrices and $i, j = R, G, B$ run over the color indices.

The transformation works in principle just as in case of the QED, it's just slightly more complex due to the eight dimensions of the $SU(3)$ generators. As in QED before, the transformation renders the free Lagrangian not invariant under $SU(3)$. We need to introduce a gauge field A^a_μ transforming according to the adjoint representation:

$$A^a_\mu \to A^a_\mu{}' = A^a_\mu - \frac{1}{g_C}\partial_\mu \alpha^a(x) - f^a_{\ bc}\alpha^b(x)A^c_\mu.$$

QCD: $SU(3)_C$

Using the quarks q and the gluons A^a_μ we can now write the Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + i\,\bar{q}_i \left(\partial\!\!\!/\, \delta_{ij} + i g_C \left(\frac{\lambda_a}{2} \right)_{ij} A\!\!\!/^a \right) q_j$$

with

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_C f^a{}_{bc} A^b_\mu A^c_\nu$$

which is different than in $U(1)$ due to the non-abelian character of $SU(3)$. A little bit more detail: The full form of the field operators can be written as:

$$q_i(x) = \sum_{\mathrm{spins}\,\lambda} \int \frac{\mathrm{d}^3 p}{\sqrt{(2\pi)^3 2p_0}} \left(a_{i\lambda}(p) u_{i\lambda}(p) e^{-ipx} + b^+_{i\lambda}(p) v_{i\lambda}(p) e^{ipx} \right),$$

analogously without the spinors u, v for the gluon field.

QCD: $SU(3)_C$: Just for completeness

What's all that stuff in the previous equation? Important are the creation and annihilation operators $a_{i\lambda}$ and $b_{i\lambda}$, obeying

$$[b_i(p), b_j^+(p')]_{\substack{+\,\text{Quarks}\\ -\,\text{Gluonen}}} = \delta_{ij}\delta^3(\vec{p} - \vec{p}\,'),$$

$$[a_\lambda(k), a_{\lambda'}^+(k')]_{\substack{+\,\text{Quarks}\\ -\,\text{Gluonen}}} = \delta_{\lambda\lambda'}\delta^3(\vec{k} - \vec{k}\,')$$

All of the above has to be done separately for $q = u, d, c, s, b, t$.

The only input parameter is $\alpha_s = \frac{g_C^2}{4\pi} \approx 0.3$ for a scale of $Q^2 \approx 1$ GeV^2

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That's it . . . a beautifully simple theory with awfully complex consequences . . .

QFD: $SU(2)_L \times U(1)_Y$ Leptonic Sector

We choose the SU$(2)_L$ doublett

$$L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L = \frac{1}{2}(1-\gamma^5)\begin{pmatrix} \nu \\ e \end{pmatrix}, \begin{array}{l} I_3 = +\frac{1}{2},\ Q = 0,\ Y = -1 \\ I_3 = -\frac{1}{2},\ Q = -1,\ Y = -1 \end{array}$$

and the singlett

$$R = e_R = \frac{1}{2}(1+\gamma^5)e,\ \ I_3 = 0,\ Q = -1,\ Y = -2$$

which transform SU$(2)_L$ according to

$$L \rightarrow L' = e^{i\alpha^a \frac{\tau_a}{2}} L, \quad R \rightarrow R' = R$$

and under U$(1)_Y$ according to

$$L \rightarrow L' = e^{i\beta^a \frac{Y}{2}} L, \quad R \rightarrow R' = e^{i\beta^a \frac{Y}{2}} R$$

QFD: $SU(2)_L \times U(1)_Y$ Leptonic Sector

Now we construct the gauge fields W^a_μ for $\mathrm{SU}(2)_L$ analogously to $SU(3)_C$ before and B_μ of $\mathrm{U}(1)_Y$ analously to the QED before. We get the covariant derivative

$$D_\mu = \partial_\mu + ig\frac{\tau_a}{2}W^a_\mu + ig'\frac{Y}{2}B_\mu .$$

Using this, we can construct the first part of the QFD Lagrangian

$$\mathcal{L}^1_{\mathrm{QFD}} = -\frac{1}{4}W^a_{\mu\nu}W^{\mu\nu}_a - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + i\overline{L}\not{D}L + i\overline{R}\not{D}R,$$

with

$$W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g\epsilon^a{}_{bc}W^b_\mu W^c_\nu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu .$$

QFD: $SU(2)_L \times U(1)_Y$ Masses

- Mass of the gauge bosons
  Now we would like to add gauge boson masses:

$$\frac{1}{2}M^2 B^\mu B_\mu$$

However, this is not invariant under $SU(2)$:

$$\to \frac{1}{2}M^2(B^\mu - \frac{1}{g'}\partial^\mu\alpha(x))(B_\mu - \frac{1}{g'}\partial_\mu\alpha(x))$$

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- Mass of the fermions

$$\begin{aligned} -m\bar{e}e &= -m\bar{e}\left(\frac{1}{2}(1-\gamma^5) + \frac{1}{2}(1+\gamma^5)\right) e \\ &= -m(\bar{e}_R e_L + \bar{e}_L e_R) \end{aligned}$$

  But only e_L and not e_R is transforming under $SU(2)$!

QFD: $SU(2)_L \times U(1)_Y$ Masses

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We have a beautiful theory of massless particles!

QFD: $SU(2)_L \times U(1)_Y$ EWSB

In order to allow masses for the gauge bosons, we introduce the Higgs doublett into the theory:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \; Y = +1 \quad \text{which is gauged like} \quad \Phi = e^{i\frac{\sigma_a \alpha^a}{2v}} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix}$$

We obtain $v = \sqrt{-\mu^2/\lambda}$ as vacuum expectation value of the field in the potential

$$V(\Phi) = \frac{\mu^2}{2}\Phi^+\Phi + \frac{\lambda}{4}(\Phi^+\Phi)^2$$

with $\lambda > 0$ and $\mu^2 < 0$, such that there is spontaneous symmetry breaking (the ground state does not obey the symmetries of the theory). ϕ^+ has to be gauged to 0 in order to render the charge operator $Q = I_3 + \frac{Y}{2}$ unbroken. Otherwise the photon acquires mass.

QFD: $SU(2)_L \times U(1)_Y$ EWSB

Using the global $SU(2)_L$ gauge transformation from before

$$L \rightarrow L' = e^{-i\frac{\sigma^a \alpha_a}{2v}} L \Rightarrow \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + \eta \end{pmatrix}$$

we obtain the following expression for the mass sector of the QFD:

$$\mathcal{L}^2_{\mathrm{QFD}} = -\sqrt{2} f (\overline{L} \Phi R + \overline{R} \Phi^+ L) + |D_\mu \Phi|^2 - V(\Phi)$$

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From where do we get the fermion masses?

$$-\sqrt{2} f (\overline{L} \Phi R + \overline{R} \Phi^+ L)$$

acts as a mass term with the Yukawa coupling parameter f determining the mass of the fermion.

QFD: $SU(2)_L \times U(1)_Y$ EWSB

The gauge boson masses are coming from

$$|D_\mu\Phi|^2 = \frac{1}{8}g^2v^2(W^a_{\mu\nu})^2 + \frac{1}{8}g'^2v^2B_\mu B^\mu - \frac{1}{4}gg'v^2B^\mu W^3_\mu$$

using

$$(W^1_\mu)^2 + (W^2_\mu)^2 = (W^1_\mu + iW^2_\mu)(W^1_\mu - iW^2_\mu) = 2W^+_\mu W^-_\mu$$

introducing the charged currents. That yields

$$\frac{1}{4}g^2v^2W^+_\mu W^-_\mu + \frac{1}{8}v^2(B^\mu, W^3_\mu)\begin{pmatrix} g'^2 & -gg' \\ -gg' & g^2 \end{pmatrix}\begin{pmatrix} B^\mu \\ W^{3\mu} \end{pmatrix}$$

We have the mass term on the $W^\pm$ already. Let's diagonalize the mass matrix of the hypercharge field B_μ and the third component of the $SU(2)_L$ gauge field W^3_μ:

$$\begin{pmatrix} A_\mu \\ Z^0_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} B^\mu \\ W^{3\mu} \end{pmatrix}$$

Now another miracle has occured: The photon field A_μ drops out of EWSB!

QFD: $SU(2)_L \times U(1)_Y$ EWSB

we have now introduced the Weinberg angle

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

From the diagonalization of the mass matrix for W_μ^3 and B_μ

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g' W_\mu^3 + g B_\mu), \quad m_A^2 = 0$$

$$Z_\mu^0 = \frac{1}{\sqrt{g^2+g'^2}}(g W_\mu^3 - g' B_\mu), \quad m_{Z^0}^2 = \frac{(g^2+g'^2)v^2}{4}$$

QFD: $SU(2)_L \times U(1)_Y$ EWSB

We also obtain the charged current and its coupling to the W_μ^+ as

$$\frac{g}{2\sqrt{2}}(\bar{\nu}_L \gamma^\mu e_L W_\mu^+ + h.c.)$$

In addition, as the first tested firm prediction of this theory, the neutral currents have been introduced ('74 November revolution: Gargamelle):

$$\frac{\sqrt{g^2+g'^2}}{4}\,(\overline{L}\gamma^\mu \tau_3 L - 2\frac{g'^2}{g^2+g'^2}\bar{e}\gamma^\mu e)Z_\mu^0, \qquad \frac{gg'}{\sqrt{g^2+g'^2}}\,\bar{e}\gamma^\mu e\, A_\mu$$

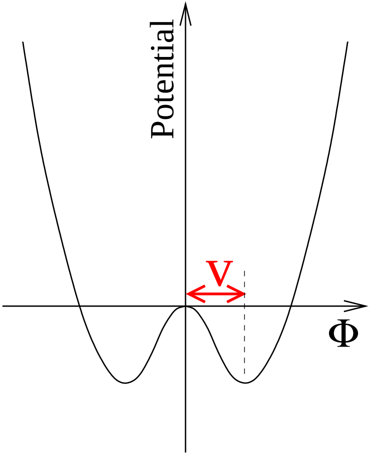
where

$$q_e = \frac{gg'}{\sqrt{g^2+g'^2}}$$

is the electromagnetic charge and $e = e_L + e_R$

This formalism has to be written for all three lepton families $\ell = e, \mu, \tau$.

QFD: $SU(2)_L \times U(1)_Y$ Properties of the Higgs



- The heavier the particle, the stronger the Higgs coupling to it (or the other way around!)
- The position of the minimum of the potential

$$V(\Phi) = \frac{\mu^2}{2}\Phi^+\Phi + \frac{\lambda}{4}(\Phi^+\Phi)^2$$

is known: Compare

$$\frac{g}{2\sqrt{2}}\bar{\nu}_L\gamma^\mu e_L W_\mu^+$$

with $V - A$ theory: $\mathcal{L}_{eff}^{V-A} \sim -\frac{G_F}{2}\ldots$

$$\left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{M_W^2} = \frac{G_F}{2} \Rightarrow v = 246\,\mathrm{GeV}$$

QFD: $SU(2)_L \times U(1)_Y$ Remarks

There are a few non-trivial observations about EWSB in the SM:

- It is not trivial that the photon field A_μ fullfills

  $$m_A = 0$$

  $$q_e \bar{e} \gamma^\mu e A_\mu$$

  (i.e. no coupling to the neutrino and the same coupling to the left and right fields) at the same time!
- All three elements of

  $$\frac{M_W}{M_Z} = \cos\theta_W$$

  can be measured independently ⇒ precision tests
- The Higgs has been introduced to give mass to the gauge bosons, but it offers an elegant way to introduce masses of the fermions, too.
- There is a self-interaction among the gauge bosons in the $-\frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu}$ term. This just pops out of the theory, it was not constructed as the gauge boson fermion interactions. Does Nature obey the SM also in this unforeseen field? ⇒ precision tests

Quarks

For the quarks, we choose the fundamental states differently for the mass and the interaction operators:

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L, \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L, \qquad u_R, \quad d_R, \quad c_R, \quad s_R, \quad t_R, \quad b_R$$

being the weak interaction eigenstates. We get the mass eigenstates using the CKM matrix:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\mathrm{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \approx \begin{pmatrix} 1 & \lambda & A\rho\lambda^3 e^{i\delta} \\ -\lambda & 1 & A\lambda^2 \\ A\lambda^3(1-\rho e^{i\delta}) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$VV^+ = 1$$

Quarks

Then the QFD of the quarks can be written in exact analogy to the leptons. We ge tadditional terms for the right-handed up-type quarks, for which we have no corresponding leptons in the SM wit hmassles sneutrinos. We use a $SU(2)$ transform of the Higgs field for the right-handed up-type quark mass terms.

$$-\sqrt{2} f_d (\bar{u}, \bar{d}') \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} d_R - \sqrt{2} f_u (\bar{u}, \bar{d}') \begin{pmatrix} -\phi^0 \\ \phi^+ \end{pmatrix} u_R .$$

Quarks

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$$-\sqrt{2}f_d(\bar{u},\bar{d}')\begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} d_R - \sqrt{2}f_u(\bar{u},\bar{d}')\begin{pmatrix}-\phi^0\\ \phi^+\end{pmatrix} u_R.$$

Input parameters to the QFD:

m_e	≈ 511 keV	m_μ	≈ 105 MeV	m_τ	$\approx 1,7$ GeV
m_u	≈ 5 MeV	m_d	≈ 5 MeV	m_s	≈ 150 MeV
m_c	$\approx 1,5$ GeV	m_b	≈ 4.7 GeV	m_t	≈ 174 GeV
m_H	$\approx$?	m_W	≈ 81 GeV	$\alpha(Q^2 \approx 0)$	$\approx 1/137$
$\sin\theta_W$	$\approx 0,23$	λ	$\approx 0,22$	ρ	≈ 0.8
A	$\approx 0,5$	δ	$\approx 0,004$		

This has to be slightly extended if neutrino masses and mixing are added.

Reading the Feynman Rules

1. Draw your Feynman diagram
2. Follow the fermion lines in opposite direction of the arrows. For each outgoing (anti)particle, write $\bar{u}(v)$, for each incoming (anti)particle $u(\bar{v})$.
3. For each incoming(outgoing) photon, write $\epsilon_\mu(\epsilon^*_\mu)$
4. For each internal line, write a propagator:
    - Fermion: $1/(\not{p} - m)$
    - Photon: $-ig_{\mu\nu}/p^2$
    - Boson: $-i(g_{\mu\nu} - p_\mu p_\nu/M^2)/(p^2 - M^2)$
5. Read the couplings from the Lagrangian:
   QED example: $\mathcal{L}_{int} = -q_e\bar{\psi}\gamma_\mu\psi A^\mu$
   denotes the coupling of an incoming fermion ψ and an outgoing fermion $\bar{\psi}$ to the photon A^μ with coupling q_e.
   In this case, we get

$$iq_e\gamma_\mu$$

   for each photon-electron vertex.

Backup Slides

History of Discoveries

- 1897 Electron discovered by J.J. Thompson
- 1899 Alpha particle discovered by Ernest Rutherford in uranium radiation
- 1900 Gamma ray (i.e. photon) discovered by Paul Villard in uranium decay.
- 1911 Atomic nucleus identified by Ernest Rutherford, based on scattering observed by Hans Geiger and Ernest Marsden.
- 1919 Proton discovered by Ernest Rutherford
- 1932 Neutron discovered by James Chadwick
- 1932 Positron discovered by Carl D. Anderson (proposed by Paul Dirac in 1927)
- 1937 Muon discovered by Seth Neddermeyer, Carl Anderson, J.C. Street, and E.C. Stevenson, using cloud chamber measurements of cosmic rays. (It was mistaken for the pion until 1946.)
- 1947 Pion discovered by Cecil Powell (predicted by Hideki Yukawa in 1934)
- 1947 Kaon, the first strange particle, discovered by G.D. Rochester and C.C. Butler
- 1955 Antiproton discovered by Owen Chamberlain, Emilio Segre, Clyde Wiegand, and Thomas Ypsilantis
- 1956 Neutrino detected by Frederick Reines and Clyde Cowan (proposed by Wolfgang Pauli in 1931 to explain the apparent violation of energy conservation in beta decay)
- 1962 Muon neutrino proved distinct from electron neutrino by group headed by Leon Lederman
- 1964 Higgs boson predicted as a result of a mechanism for electroweak symmetry breaking proposed by Peter Higgs (remains hypothetical as of 2005, but widely expected to be found at the Large Hadron Collider at CERN in the early 2010s)
- 1969 Partons (internal constituents of hadrons) observed in deep inelastic scattering experiments between protons and electrons at SLAC; this was eventually associated with the quark model (predicted by Murray Gell-Mann and George Zweig in 1963) and thus constitutes the discovery of the up quark, down quark, and strange quark.
- 1974 J/Ψ particle discovered by groups headed by Burton Richter and Samuel Ting, demonstrating the existence of the charm quark (proposed by Sheldon Glashow, John Iliopoulos, and Luciano Maiani in 1970)
- 1975 Tau lepton discovered by group headed by Martin Perl
- 1977 Upsilon particle discovered at Fermilab, demonstrating the existance of the bottom quark (proposed by Kobiyashi and Maskawa in 1973)
- 1979 Gluon observed in three jet events at DESY.
- 1983 W and Z bosons discovered by Carlo Rubbia, Simon van der Meer, and the CERN UA-1 collaboration (widely expected, predicted in detail by Sheldon Glashow, Abdus Salam, and Steven Weinberg in the 1960s)
- 1995 Top quark discovered at Fermilab
- 2000 Tau neutrino proved distinct from other neutrinos at Fermilab